Hong Kong Mathematics Olympiad (2002 – 2003)
Final Event 1 (Individual)
香港数学竞赛 (2002 – 2003)
决赛项目 1 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 设 P 是 $3^{2003}\times5^{2002}\times7^{2001}$ 的个位数。求 P 的值。

Let P be the units digit of $3^{2003}\times5^{2002}\times7^{2001}$. Find the value of P.

2. 若方程 $\left(x^2-x-1\right)^{x+P-1}=1$ 有 Q 个整数解，求 Q 的值。

If the equation $\left(x^2-x-1\right)^{x+P-1}=1$ has Q integral solutions, find the value of Q.

3. 设 x，y 为实数且 $xy=1$。若 $\dfrac{1}{x^4}+\dfrac{1}{Qy^4}$ 的最小值是 R，求 R 的值。

Let x, y be real numbers and $xy=1$. If the minimum value of $\dfrac{1}{x^4}+\dfrac{1}{Qy^4}$ is R, find the value of R.

4. 设 x_R、x_{R+1}、…、x_K $(K>R)$ 为 $K-R+1$ 个不相同的正整数且 $x_R+x_{R+1}+\cdots+x_K=2003$。若 S 是 K 的最大可能的值，求 S 的值。

Let x_R, x_{R+1}, …, x_K $(K>R)$ be $K-R+1$ distinct positive integers and $x_R+x_{R+1}+\cdots+x_K=2003$. If S is the maximum possible value of K, find the value of S.

Hong Kong Mathematics Olympiad (2002 – 2003)

Final Event 2 (Individual)

香港数学竞赛 (2002 – 2003)

决赛项目 2 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若一个两位数 P 的 50 次方是一个 69 位数，求 P 的值。(已知 $\log 2 = 0.3010$，$\log 3 = 0.4771$，$\log 11 = 1.0414$)

If the 50^{th} power of a two-digit number P is a 69-digit number, find the value of P. (given that $\log 2 = 0.3010$, $\log 3 = 0.4771$, $\log 11 = 1.0414$)

2. 方程式 $x^2 + ax - P + 7 = 0$ 的根是 α 和 β；而方程式 $x^2 + bx - r = 0$ 的根是 $-\alpha$ 和 $-\beta$。若方程式 $(x^2 + ax - P + 7) + (x^2 + bx - r) = 0$ 的正根是 Q，求 Q 的值。

The roots of the equation $x^2 + ax - P + 7 = 0$ are α and β, whereas the roots of the equation $x^2 + bx - r = 0$ are $-\alpha$ and $-\beta$. If the positive root of the equation $(x^2 + ax - P + 7) + (x^2 + bx - r) = 0$ is Q, find the value of Q.

3. 已知 ΔABC 为一等腰三角形，$AB = AC = \sqrt{2}$ 及 BC 上有 Q 个点 D_1, D_2, … , D_Q。设 $m_i = AD_i^2 + BD_i \times D_iC$。若 $m_1 + m_2 + m_3 + \cdots + m_Q = R$，求 R 的值。

Given that ΔABC is an isosceles triangle, $AB = AC = \sqrt{2}$ and D_1, D_2, … , D_Q are Q points on BC. Let $m_i = AD_i^2 + BD_i \times D_iC$. If $m_1 + m_2 + m_3 + \cdots + m_Q = R$, find the value of R.

4. 有 2003 个袋从左至右排列。已知最左面的袋装有 R 个球，而且每 7 个相邻的袋共装有 19 个球。若最右面的袋有 S 个球，求 S 的值。

There are 2003 bags arranged from left to right. It is given that the leftmost bag contains R balls, and every 7 consecutive bags contain 19 balls altogether . If the rightmost bag contains S balls , find the value of S .

Hong Kong Mathematics Olympiad (2002 – 2003)
Final Event 3 (Individual)
香港数学竞赛 (2002 – 2003)
决赛项目 3 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知 $\begin{cases} wxyz=4 \\ w-xyz=3 \end{cases}$ 且 $w>0$。若 w 的解是 P，求 P 的值。

Given that $\begin{cases} wxyz=4 \\ w-xyz=3 \end{cases}$ and $w>0$. If the solution of w is P, find the value of P.

2. 设 $[y]$ 表示小数 y 的整数部分，如 $[3.14]=3$。若 $\left[\left(\sqrt{2}+1\right)^P\right]=Q$，求 Q 的值。

Let $[y]$ represents the integral part of the decimal number y. For example, $[3.14]=3$. If $\left[\left(\sqrt{2}+1\right)^P\right]=Q$, find the value of Q.

3. 已知 $x_0y_0\neq 0$ 及 $Qx_0^2-22\sqrt{3}x_0y_0+11y_0^2=0$。若 $\dfrac{6x_0^2+y_0^2}{6x_0^2-y_0^2}=R$，求 R 的值。

Given that $x_0y_0\neq 0$ and $Qx_0^2-22\sqrt{3}x_0y_0+11y_0^2=0$. If $\dfrac{6x_0^2+y_0^2}{6x_0^2-y_0^2}=R$, find the value of R.

4. 四边形 $ABCD$ 两对角线 AC 和 BD 互相垂直。已知 $AB=5$，$BC=4$，$CD=R$。若 $DA=S$，求 S 的值。

The diagonals AC and BD of a quadrilateral $ABCD$ are perpendicular to each other. Given that $AB=5$, $BC=4$, $CD=R$. If $DA=S$, find the value of S.

Hong Kong Mathematics Olympiad (2002 – 2003)

Final Event 4 (Individual)

香港数学竞赛 (2002 – 2003)

决赛项目 4 (个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 如果 9 位数 $\overline{32x35717y}$ 是 72 的倍数，$P = xy$，求 P 的值。

Suppose the 9-digit number $\overline{32x35717y}$ is a multiple of 72 and $P = xy$, find the value of P.

2. 已知三条直线 $4x + y = \frac{P}{3}$，$mx + y = 0$ 和 $2x - 3my = 4$ 不能构成一个三角形。若 $m > 0$ 及 Q 是 m 的最小可能的值，求 Q。

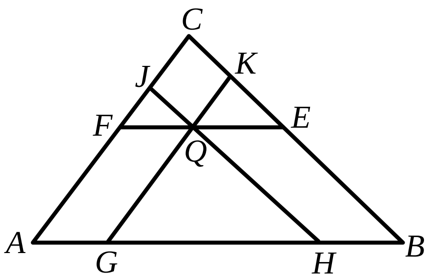
Given that the lines $4x + y = \frac{P}{3}$, $mx + y = 0$ and $2x - 3my = 4$ cannot form a triangle. Suppose that $m > 0$ and Q is the minimum possible value of m, find Q.

3. 已知 R，x，y 及 z 是整数且 $R > x > y > z$。若 R，x，y 及 z 满足方程 $2^R + 2^x + 2^y + 2^z = \frac{495Q}{16}$，求 R 的值。

Given that R, x, y, z are integers and $R > x > y > z$. If R, x, y and z satisfy the equation $2^R + 2^x + 2^y + 2^z = \frac{495Q}{16}$, find the value of R.

4. 在图一，ΔABC 内任选一点 Q 作三条分别平行于角边的直线，其中 $FE \,//\, AB$，$GK \,//\, AC$ 及 $HJ \,//\, BC$。ΔKQE，ΔJFQ 及 ΔQGH 的面积分别是 R， 9 及 49。若 ΔABC 的面积是 S，求 S 的值。

In Figure 1 , Q is an interior point of ΔABC . Three straight lines passing through Q are parallel to the sides of the triangle such that $FE \,//\, AB$, $GK \,//\, AC$ and $HJ \,//\, BC$. Given that the areas of ΔKQE , ΔJFQ and ΔQGH are R , 9 and 49 respectively. If the area of ΔABC is S , find the value of S .



图一

Figure 1